Complex Network Characteristics and Invulnerability Simulating Analysis of Supply Chain

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Abstract—To study the characteristics of the complex supply chain, an invulnerability analysis method based on the complex network theory is proposed. The topological structure and dynamic characteristics of the complex supply chain network were analyzed. The fact was found that the network is with general characteristics of the complex network, and with the characteristics of small-world network and scale-free network. A simulation experiment was made on the invulnerability of the supply chain network in different attack modes, of which the outcome demonstrates that the supply chain network is not only with the robustness in random failure, but also with the vulnerability to deliberate assault. In addition, the outcome also shows the network reliability depends on the stability of critical nodes (i.e. core enterprises). As a result, it is vital to ensure this kind of enterprise operating normally and efficiently.

Index Terms—complex network characteristics, supply chain, invulnerability, failure in random, deliberate assault

I. INTRODUCTION

A large number of complex system in the nature and human society can be described as various networks [1,2]. A typical network is to be constituted with nodes and edges connecting two nodes, among them nodes represent different individuals in real system, and edges express the relationship between individuals. For example, human society is the network joined by persons and kinds of social relations. A supply chain as a social system can be better described as the complex network with which it can be analyzed and modeled. At present the main study of the complex supply chain network is by the scholars headed by Dirk Helbing [3].They think that the supply chain network is a complex adaptive system. They found the information amplification effect of supply chain management, namely the bullwhip effect, which can be weakened by the excellent supply chain topology structure, and the excellent supply chain topology structure can also increase network resistance to the attack. Christian Kuhnert [4] reached a conclusion that the city supplies is a scale-free distribution network, in which there are minority key nodes playing an important role on distributing and scheduling of goods. Thadakamalla and others [5] analyzed the survivability of complex supply chain network. Huang Xiao Yuan and others [6] reviewed the robust of supply chain network. In sum, the previous research is less on the invulnerability of complex supply chain network. As a complex network, the supply chain system mostly lacks the ability to resist uncertainty and even can't resist risk. Therefore, the study on the invulnerability of complex supply chain network has very important significance to improve the performance and stability of supply chain system. The study analyzed the topology structure of supply chain network, modeled invulnerability and made simulating experiment, got the forms and features of complex supply chain network invulnerability with different interference.

II. COMPLEX NETWORK CHARACTERISTICS OF SUPPLY CHAIN

A. Structure of Supply Chain Network

An ordinary supply chain network has a manufacturer as the core enterprise, whose upstream is the supply system composed of all suppliers, whose downstream is the sales system composed of distributors and retailers. The structure of a supply chain network is shown in Fig.1.

A complex supply chain network is composed by the vertexes and edges, the vertexes represent all member enterprises, the edges represent various flows between enterprises. Dynamic connections between nodes enterprises are created by logistics, information flows, capital flows, etc.

Since the joint function of network members gathered is greater than the sum of each enterprise separate function, in order to prevail over the others, enterprises
must gather themselves to form the supply chain to adapt to the changeful market environment. With each node enterprise interaction, the dynamic evolution model of the whole supply chain emerges, and the model will promote restructuring and replacement of supply chain. Meanwhile, there is also the interactions between supply chains, supply chains connect each other, interact with each other with the way of complex coupling and influence each own behavior pattern [7-9].

B. Supply Chain Network Topology Characteristics

In supply chain network topology structure, the vertexes represent the enterprises participating in supply chain activities and are classified with their positions in the supply chain (e.g. suppliers, manufacturers, distributors and customers, etc); the edges represent the relationships between enterprises in corresponding vertexes (i.e. the transaction activity or more general called supply and demand relation); one vertex’s degree says the adaptability of the vertex enterprise (or called comprehensive competitive power), is often the integrated performance of various performance index, such as the supplier’s delivery ability; the weight of edge says the number of transaction of the corresponding node in a fixed period.

Only a few indexes mainly affect the supply chain network structure characteristics and dynamic characteristics. These indexes reflect the complex characteristics of supply chain network in different aspects [10-12].

Average path length (L): Average path length says the time of delivery of the products in a supply chain. In the supply chain network with N nodes enterprises, the transportation distance $d_{ij}$ between the node enterprise $i$ and the node enterprise $j$ is defined as the number of edges in the shortest path connecting this two nodes enterprises; then

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}$$

(1)

Average clustering coefficient ($C$): Average clustering coefficient refers to the degree of mutual exchange between nodes enterprises in a supply chain. Considering the node enterprise $i$ has $k_i$ edges connecting with other nodes enterprises, if the nearest point of the initial node is a part of the entire group, there are $k_i(k_i-1)/2$ edges connecting between them. The specific value of $E_i$, the actual number of edges between $k_i$ nodes, and $k_i(k_i-1)/2$, the total number of edges, is the value of clustering coefficient of the node $i$:

$$C_i = \frac{2E_i}{k_i(k_i-1)}$$

(2)

The clustering coefficient $C$ of the supply chain network with $N$ nodes is the average value of clustering coefficients of all $i$ nodes, namely

$$C = \frac{1}{N} \sum_{i=1}^{N} C_i$$

(3)

In a supply chain network, the clustering coefficient $C$ larger shows nodes enterprises trade with others frequently.

Degree and degree distribution: The degree of a node in supply chain network refers to the number of edges connecting the node enterprise. Degree is a main parameter describing network local characteristics. Not all nodes enterprises in a supply chain network are with the same degree. Degree distribution refers to the rule of the probability $P(k)$ of the node enterprise with degree $k$ appearing in a supply chain network changing with the change of node degree $k$. Degree distribution can also be approximately understood for the proportion of the number of nodes enterprises with degree $k$ to the total number $N$ of all nodes in the supply chain network, namely

$$P(k) = \text{frequency}(k) / N$$

(4)

In which, frequency$(k)$ for the number of nodes enterprises with degree $k$ in the supply chain network.

The degree distribution function reflects the macro statistical characteristics of the supply chain network.

In a supply chain network, each node enterprise by reorganization and transformation, establishes distribution center, reduces transportation distance, realizes just-in-time delivery, delivers product to the customer in the shortest possible time, so that keeps advantage in the fierce competition. Thus it can be seen that a complex supply chain network is with smaller average path length. Therefore, a complex supply chain network has small-world network characteristics [13].

Along with the rapid development of modern information technology and popularization of Internet,
the connection of nodes enterprises is closer and the
exchange more frequently by information sharing,
namely a supply chain network is with high clustering
coefficient. The node degree of a complex supply chain
network obeys the power-law distribution, as shown in
Fig.2. That is, the relationship between the number of
nodes with a certain degree and the certain degree can be
expressed approximately with a power function. The
power function is \( P(k) \propto k^{-\gamma} \), in which, \( \gamma \) is the power
exponent, the power function curve is a curve relatively
slowing down, that can make nodes enterprises with
larger degree, i.e. core enterprises, exist in supply chain
network. These core enterprises are called Hub points of
supply chain network\[14\]. Applying information
technology, these core enterprises establishes close
contact with other nodes enterprises, builds the supply -
production - distribution system around themselves.
Therefore, most nodes enterprises in a complex supply
chain network are with very low degree, but core nodes
enterprises are with very high degree, and core nodes
enterprises plays a leading role on the operation of the
supply chain network. This shows that a complex supply
chain network is with scale-free characteristics[15,16].

III. THE ROBUSTNESS AND VULNERABILITY OF COMPLEX
SUPPLY CHAIN NETWORK

Globalization makes the modern production process
include various intermediate links such as material
purchase, production, distribution and transportation,
many enterprises involved in, and constitute the complex
supply chain network. Along with the globalization
depening, the enterprises in the supply chain network are
more and more, the structure of the supply chain network
is more and more complex. The wrong of any link will
impact the whole supply chain hugely. The robustness
of the supply chain, is the ability of the system to keep
the whole supply chain efficiency optimal and smooth
operation function under being disturbed uncertainly by
the internal operation and external emergency event.

After the uncertainty attack, in generally, nodes
enterprises cannot trade with other nodes enterprises in
supply chain network, for example transport routes
interrupted, network paralyze. And that makes the
connection of nodes enterprises be interrupted, the nodes
enterprises is not able to match the network attacked
themselves, which causes production stagnation, business
failures, etc.

Assume given a supply chain network, a enterprise
node is removed from the network every time, all the
edges connecting this enterprise node is also removed at
the same time, and which may make some paths between
other enterprises nodes in the supply chain network
interrupt. If there are multiple paths between node \( i \) and
node \( j \), the interrupt of some these paths may make the
distance between the two nodes \( d_{ij} \) increase, and the
average path length \( L \) of the whole supply chain network
also increase. And if all the paths between node \( i \) and
node \( j \) are interrupted, the two nodes will no longer
connect each other. The process of supply chain network
enterprise nodes removal is shown in Fig. 3.

At no attack, the distance between enterprise node \( A \)
and enterprise node \( B \) in the supply chain network is 2,
but with the two enterprise nodes being removed, the
distance between the enterprise node \( A \) and enterprise
node \( B \) increases to 6. At the same time, the supply chain
network is split into five fraction systems.

The enterprise node is the basic unit constituting the
whole supply chain network. The attack to the enterprise
node will inevitably make the structure and dynamic
properties of entire network change, ultimately affect the
stability of the supply chain network. Failure in random is
to remove some nodes at random, and deliberate assault
is to intentionally remove the enterprises nodes with high
degrees, which plays a critical role on the network
structure. The results of the study on the supply chain
network topology structure shows that the network
Invulnerability depends on the network degree
distribution form. The supply chain network is with
scale-free network characteristics, and with high
robustness to random failure. Because the supply chain
network degree distribution is with extreme heterogeneity, the degree of most nodes is very small, and
the degree of a few nodes relatively large. It is the heterogeneity of supply chain network to make the network with a high vulnerability to deliberate assault. As long as consciously to remove the network enterprise node with largest degree, the entire supply chain of network connectivity will be large effected.

IV. COMPLEX SUPPLY CHAIN NETWORK INVULNERABILITY MODEL

A. Definition 1

Under the restriction of a certain connectivity, the biggest remove scale of nodes or edges which a complex supply chain network can bear is called the invulnerability degree or stability degree of the network.

The network invulnerability is related to slowing performance of some edges or nodes in network after attack. The network invulnerability problem is changed to a seepage problem in a generalized random map. The complex network invulnerability is analysed with the seepage theory to get the critical value of collapse nodes removed proportion after the network attacked:

\[ f_c = 1 - \frac{1}{k - 1} \]

in which,

\[ k \] for the node degree,

\[ k = \langle k^2 \rangle / \langle k \rangle \] for the expectations of the node degree \( k \).

B. Definition 2

Assume \( G(N, p) \) for the collection of supply chain networks with \( N \) nodes. When a node \( i \) in the supply chain network set \( G(N, p) \) is randomly removed and causes the supply chain network performance declining, the supply chain network vulnerability (namely invulnerability failure) can be expressed as:

\[ V(G) = \frac{1}{N} \sum_{i \in G} |E(G) - E(G/\{i\})| \]

in which,

\( N \) for the total of nodes,

\[ E(G) = \frac{1}{N(N-1)} \sum_{i,j \in G, i \neq j} \frac{1}{d_{ij}} \] for the efficiency of the network,

\[ d_{ij} \] for the shortest distance between the node \( i \) and node \( j \).

C. Definition 3

Assume \( G(N, p) \) for the collection of supply chain networks with \( N \) nodes. When nodes with high connection degree in the supply chain network set \( G(N, p) \) are attacked or removed, and causes the whole supply chain network performance decline sharply, the vulnerability of supply chain networks after being intentionally attacked is expressed as:

\[ V(G) = \max\{E(G) - E(G/\{i\})|i \in G\} \]

D. Definition 4

Assume \( G(N, p) \) for the collection of supply chain networks with \( N \) nodes, then the indicator function of the supply chain network vulnerability is with the following characteristics:

First, for a given \( G(N, p) \), \( V \) is unchanged;

Second, if \( G' \) is got by adding a side to \( G \), then \( V(G') \leq V(G) \);

Third, \( V(G) \) can be calculated in the formula related to the number of vertexes in supply chain network set \( G(N, p) \).

According to the definition 4, get the supply chain networks invulnerability failure function:

\[ V(G) = \exp\left\{ \frac{K}{N} - \frac{k}{N} - \left|e - 2 + \frac{2}{N}\right\} \right\} \]

in which, \( K \) is the degree of highest degree node and \( k \) is the minimum degree of nodes in the supply chain network, \( e \) for the total of edges connecting nodes, \( N \) for the total of nodes. Because the supply chain networks with marked different vulnerability can't be distinguished intuitively in (8), (8)transforms to:

\[ V(G) = \exp\left\{ \frac{\omega}{N} + \frac{k}{N} - \left|e - 2 + \frac{2}{N}\right\} \right\} \]

in which \( k \) is the degree of the node \( i \), \( \omega \) is the degree deviation of nodes.

V. SIMULATION ALGORITHM

A. Vulnerability Simulation of the Complex Supply Chain Network in Random Failure

The vulnerability simulation algorithm of the complex supply chain network in random failure is as follows:

Step1: When the beginning time \( t = 0 \), assume the complex supply chain network composed by \( N \) nodes;

Step2: When \( t = 1 \), there is \( N-1 \) nodes in the supply chain network in addition to node \( o \), random select \( N_0 \) nodes from \( N-1 \) nodes and connect them with node \( o \) to constitute the initialization supply chain network;

Step3: Random choose any two nodes \( i \) and \( j \) in the initialization supply chain network, set \( d_{ij} \) for the shortest distance between node \( i \) and node \( j \), and remove a node of the two, if \( d_{ij} = 0 \), then the edge connecting node \( i \) and node \( j \) is broken;

Step4: Repeat Step3 until all edges connecting all nodes in the supply chain network are broken.

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B. Vulnerability Simulation of the Complex Supply Chain Network under Deliberate Attack

The thought of the complex supply chain network collapsing under deliberate attack is as following:

First, fracture the edge connecting the two nodes with highest degree in the network, count the path distance and scale of the network;

Second, then recalculate the degree of each node, and refracture the edge connecting the two nodes with biggest degree in network again, recount the path distance and scale of the network;

Third, so repeatedly, until all edges connecting all nodes in network are broken.

Its algorithm simulation procedure is as follows:

Step1: Initialize the path distance of the supply chain network, cause 0

$$L_0 = \text{scale of } NS$$

Step2: Calculate the product of two nodes degrees in supply chain network, that is

$$d_{ij} = \begin{cases} \text{d}_i \cdot \text{d}_j, & m_{ij} = 1 \\ 0, & m_{ij} \neq 1 \end{cases}$$

in which, $$d_i = \sum_{k=1}^{N} m_{ik} d_j = \sum_{k=1}^{N} m_{jk}$$.

Step3: Matrix $$D = (d_{ij})_{N \times N}$$, $$d_{ij}$$ for the shortest distance between node $$i$$ and node $$j$$, cause

$$d_{ik} = \max(\max(D))$$

and make the elements $$m_{ki} = 0, m_{ik} = 0$$ in the $$M$$, get a new weight matrix

$$M^{(t)}, (t = 1, 2, \cdots)$$

if $$d_{ij} = 0$$, stop;

Step4: If $$d_{ij} \neq 0$$, as some nodes break their connecting other nodes edges, a new supply chain network can be got, according to Floyd’s shortest path method, recalculate $$d_{ij}$$:

First, initialize,

$$D = (d_{ij})_{N \times N}, (i, j = 1, 2, \cdots, N)$$

in which,

$$d_{ij} = \begin{cases} m_{ij}, & m_{ij} \neq 0 \\ \infty, & m_{ij} = 0 \end{cases}$$

Second, make the weight matrix

$$D^{(0)} = D$$

Calculate

$$D^{(s)} = (d_{ij}^{(s)})_{N \times N}, (s = 1, 2, \cdots, N)$$

in which,

$$d_{ij}^{(s)} = \min[d_{ij}^{(s-1)}, d_{ks}^{(s-1)} + d_{kj}^{(s+1)}]$$

Third, After $$t$$ moments, the supply chain network scale becomes $$S^{(t)}$$, then, scratch the elements which entire row or entire column are of $$\infty$$ in the weight matrix $$D^{(s)}$$, calculate the average distance of the new supply chain network:

$$L_t = \frac{\sum_{i=1}^{t} \sum_{j=1}^{t} d_{ij}^{(s)}}{S^{(t)}(S^{(t)} - 1)}$$

Step5: Repeat Step2 ~ Step4 until all edges connecting all nodes in the supply chain network broken.

Draw Fig. 4, Fig. 5 as below to explain intuitively the average shortest distance of the complex supply chain network and the relationship between the network scale and the number of failure edges in the network.

The two figures both adopt relative coordinates, the abscessas both denote the count of relative failure supply chain network connections, say the ratio of the number of failure edges and the number of initial edges in the supply chain network, the ordinate in Fig.4 expresses the average short distance of the initial supply chain network, the supply chain network scale.

Fig. 4 shows, contrasted with the supply chain network in random failure situation, the average shortest distance of the supply chain network increases rapidly when the same amount of nodes enterprises are attacked deliberatly. When the ratio of fracture nodes in supply chain network $$f = 0.55$$, the average shortest distance of the supply chain network under deliberate attack reaches the maximum $$L = 3.15$$, while the average shortest distance in random failure situation is $$L = 0.95$$. In addition, in the initial stage of deliberate attack, the average shortest distance of the supply chain network increases with the increase of nodes enterprises attacked, reaches the maximum $$L = 3.15$$, then declines rapidly. This is because being attacked the supply chain network is divided into several blocks, causes the total scale of the

Figure 4. The change curve of supply chain network average distance

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supply chain network smaller, and the average shortest distance of the whole network also smaller. When the deliberate attack makes the fracture nodes ratio \( f \) increase to about 0.71, the average shortest distance of the network turns to \( L = 0 \), finally makes the supply chain network collapse.

Fig. 5 shows, if the supply chain network is attacked deliberately, the number of node enterprises attacked is linearly digressive to the supply chain network scale. When in supply chain network the fracture nodes ratio \( f = 0.50 \), the supply chain network scale at that time is only less than half of its initial scale, then the supply chain network closes to collapse. When the ratio of fracture nodes in supply chain network in random failure is half of its initial scale, the scale after fracture is similar to the initial scale of supply chain network. If \( f = 0.57 \), the scale of the supply chain network starts to diminish until the entire network collapse.

It’s known from the above analysis that the supply chain network with scale-free characteristics is with strong invulnerability to random failure, that comes from its special topology structure.

Its essence is to remove the nodes enterprises (i.e. the non-core enterprises with little correlation to other enterprises) in supply chain network randomly. Because the quantity of these nodes enterprises is far more than of Hub enterprises (i.e. the core enterprise with great correlation to other enterprises), and the non-core enterprises have only a small number of link points, removing non-core enterprises will not produce major effect on the topology structure of the supply chain network. But this topology structure depends particularly on the Hub enterprises. When the Hub enterprises are attacked deliberately, the supply chain network will emaciate. So long as remove a few Hub enterprises, the entire supply chain network will collapse[17].

VI. EXAMPLE OF THE COMPLEX SUPPLY CHAIN NETWORK ANALYSIS

Construct a supply chain network with \( N = 60 \) nodes and 2280 edges. All suppliers, manufacturers, distributors, customers are the supply chain network nodes. The transaction activities between enterprises form the supply chain network edges, which represent the relationships between enterprises in corresponding vertexes. The enterprises’ comprehensive competitive powers are the degrees of nodes. The invulnerability analysis of the supply chain network is as below. As a comparison, take the connected coefficient threshold \( \theta = 0.30 \) and \( \theta = 0.65 \), to calculate the average path length, the clustering coefficient and the invulnerability degree of the network at various attacks. The results is shown in Table I.

Table I. shows, when the threshold \( \theta = 0.30 \), the supply chain network is with smaller average path length, accords with the small-world network characteristics. That explains the supply chain network can shorten the distance of transporting products in nodes, and deliver products to customers in the shortest possible time.

When the threshold \( \theta = 0.65 \), the supply chain network is with larger clustering coefficient, is with obvious scale-free network characteristics. That shows the transaction of the supply chain network around the core nodes enterprises is more frequently. At \( \theta = 0.30 \) and \( \theta = 0.65 \), the supply chain network is with good stability at random attack. While \( \theta = 0.65 \), the whole supply chain network is with very bad invulnerability at deliberate attack. That is because the network topology structure is scale-free characteristics, in fact, as long as the core enterprises is removed from the supply chain network, the whole supply chain network is basically paralyzed. Relatively, at \( \theta = 0.30 \), the supply chain network is with good stability at deliberate attack, which because when \( \theta = 0.30 \), the supply chain network topology structure is with the small-world network characteristics.

VII. CONCLUSION

The complexity of the supply chain system makes the supply chain be with the general characteristics, small-world characteristics and scale-free characteristics of the complex network. To damage the important nodes in the supply chain network such as core enterprises maliciously, will influence on the entire supply chain tremendously. Therefore, the emergency plan of the core enterprises in supply chain network shall be made at first. The core enterprises in the supply chain network shall choose multiple suppliers nodes, because the raw material
supply fluctuation will influence not only on the core enterprises own business, but also on the rest other nodes enterprises in the supply chain, and cause the overall effectiveness of the supply chain network reduce, even cause the supply chain network collapse. Next, the violent changes of core enterprises’ policy and personnel shall be avoided to maintain the stability of the supply chain network. The serious change of core enterprises will affect nodes enterprises associated, and affect the whole supply chain network.

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